Animals are born to move

When we watch children taking their first steps, a batsman hitting a home run, a pianist playing Mozart, birds diving on prey, bees flitting from flower to flower, we cannot but wonder: How do they do that? Understanding movement is central to understanding development. Without movement we - by which I mean the animal kingdom - would not be able to eat, avoid harm, reproduce or communicate by sound, gesture or facial expression. We would not be able to perceive, since perception is an active process. Consequently we would not be able to think, because there would be nothing to think about. We would not even be able to breathe or pump nutrients around the body. In short we would be dead.

How is movement controlled and how does the ability develop? I shall first consider basic principles underlying animal movement. Next I shall outline a theory (General Tau Theory) of movement guidance based on those principles. Finally, I shall describe experiments testing the theory and discuss applications of the theory in the study, and measurement, of development of movement control.
Principles of animal movement

James J Gibson and Nicholai Bernstein never met and knew little or nothing about each other’s work. However, they came to very similar conclusions about the nature of animal movement. Separately they pioneered the field of perception and movement control and laid firm foundations for future research. Gibson approached the problem more from the perceptual angle (Gibson 1966), Bernstein more from the movement side (Bernstein 1967). From their work the following five general principles of animal movement may be distilled.

1. **Movement requires perceptual guidance.** The reason why this must be so is that movements are brought about not simply by muscular forces but also by external forces such as gravity and friction. Because the external forces are not wholly predictable, they could deviate a movement from its intended course. Therefore, the progress of a movement needs to be monitored perceptually to enable appropriate muscular adjustments to be made. One or more of the perceptual systems may be involved in the monitoring. The articular proprioceptive system, comprising sensors in the joints, muscles and skin, is constantly active during all movements. The vestibular system is active whenever head movement is part of the action, which it normally is. Hearing is active in movement monitoring whenever controlled sounds are being produced, as when speaking, singing, playing a musical instrument and guiding movement around the world using echolocation, whether as bat, dolphin or human. Vision can be active in a multitude of ways in guiding movement. However, one cannot be looking everywhere at once, and to pick up detailed information you need to move your gaze around in an efficient way. Learning to drive, for example, is very much about learning where to look and when. Vision also appears to act as an overseer of the other perceptual systems, keeping them mutually in tune (Lee 1978).

2. **Movement requires intrinsic guidance.** An animal fashions movements to its purpose and so movement must also be guided intrinsically. When singing, for example, the music that guides the voice comes from within the singer - though, as I’ve just mentioned, hearing is required to monitor the voice to make sure its doing what the inner
music directs. Running is another example. The running style comes from within the runner, while the eyes guide the progress across the ground.

3. **Movements are prospectively guided.** They flow ahead in time like a melody. This has to be so because an animal has limited power available for making a movement. Therefore, if it does not manage its power resources prospectively it could end up not having sufficient power available to complete a movement properly. This could have dire consequences if an animal runs out of braking power when trying to stop at a cliff edge.

4. **Movement information embraces the future.** For movements to be guided prospectively, the information guiding the movement, whether perceptual or intrinsic, must allow adequate extrapolation of the movement into the future. Therefore, the information must have a temporal structure that extends it forwards beyond the immediate present.

5. **Movement guidance is simple, rapid and reliable - and probably follows universal principles.** That movement guidance is rapid and reliable is evident from watching the behaviour of any animal. The fact that animals with very small nervous systems, such as insects, perform movements with a precision comparable to our own suggests common simple underlying principles of movement guidance.

**Information guiding hands and other effectors**

Watching a slow motion film of a fielder catching a fast cricket ball with one hand while diving over the ground brings out two important points. First, the body can move in an indefinite number of ways when directing an effector (the hand in this case) towards something. Even in everyday activities this is the rule rather than the exception. Consider for example picking up a coffee cup from a desk. This can be done with equal facility when stepping over to the desk, or when swivelling around on a desk chair, or when simply reaching across the desk. In short, reaching is about controlling the movement of the hand relative to the object of the reach. Depending on the circumstances, this can require a variety of different forms of body movement.
The second point concerns the connection between the hand and the object. When watching a fielder catching a ball one can get the impression that the ball is physically connected to the hand, even before the catch is made. It is as if hand and ball are connected by invisible elastic that draws them together. There is, in fact, a physical connection between the hand and the ball before contact is made. It is not, of course, a material connection like a piece of elastic. Rather it is an informational connection, more like that between an operator and a radio-controlled model plane. In general, the information that prospectively guides movement is obtained through several perceptual systems. For the fielder, these at least include vision and the articulatory system of sensors in joints, muscles and skin. For a bat catching an insect on the wing, echolocation is used instead of vision. But whatever the perceptual systems involved, a central component is the information about the changing motion-gap between an effector and its goal (hand and ball for the fielder, wing and insect for the bat) that enables the gap to be prospectively controlled. The motion-gap may be proprio-specific, between an effector and part of the body, as when putting food in the mouth; or it may be extraproprio-specific, between an effector and an external object or surface, as in the ball-catching example (Lee 1978).

As a general rule, we may think of an effector as anything that is controlled to a goal. It may be a hand as in grasping, a foot as in securing footing, a mouth as in seizing food. Or it may be a tool controlled by a person. When manipulating a computer mouse one reaches with the cursor, not the hand. When holding a laser pointer one reaches with the laser beam. When using a remote-controlled surgical instrument one reaches with the remote instrument. In all these examples, the body movements are quite different. Therefore, the essence of moving an effector to its goal does not reside in the pattern of limb movements that moves the effector. Rather it lies in the form of closure of the motion-gap between the effector (hand, foot, cursor, beam, instrument) and the goal.

Controlling the closure of the motion-gap between an effector and its goal, as when reaching, is a perceptuo-motor act. However, most theories of reaching have primarily addressed just the motor aspects. They have been basically concerned with explaining the dynamics of reaching with the arm and hand, in terms of mathematical models of the physico-neural structure of the musculature (Feldman & Levin 1995; Flanagan et al.
1993; Bizzi et al. 1992). The theories do assume that movement of the arm and hand is directed by perceptual information - by, e.g., shifting the limb's 'equilibrium point' (see Flanagan et al 1993 for a review of different forms of the 'equilibrium point hypothesis') - but no explanation of the perceptual component is considered in the theories. However, it is essential that the perceptual component is taken into account when trying to understand control of reaching or any other movement. For example, visual information picked up during reaching can guide the limb and adjust for motion of the goal (Flash 1990; Georgopoulos et al. 1981; Pellison et al. 1986; Soechting and Lacquaniti 1983; van Sonderjen et al. 1989).

In short, perceptual information is part and parcel of an act. Thus, if a motion-gap variable, such as its size, is being controlled, there must be a perceptual information variable (or variables) that specifies the value of the motion-gap variable. Conversely, if there is no perceptual information variable that specifies the value of a particular motion-gap variable then it cannot be controlled. Consider, for example, moving a laser spot to a goal position across a wall that is an uncertain distance away. This can be done quite smoothly and accurately, even though there is no perceptual information variable specifying the size of the motion-gap on the wall between the current position of the laser spot and the goal. Therefore it cannot be the size of the motion-gap that is being controlled in this situation, but some other measure, X. To be sure, if, in another situation, there were perceptual information variables specifying both X and the size of a motion-gap, then the size might be controlled in this case. However, an explanation of control that applies to all situations is to be preferred to a set of ad hoc explanations. Therefore, let us see what General Tau Theory has to offer by way of providing a universal control variable for all situations.

### General Tau Theory

To summarise thus far, I have argued that an adequate theory of guidance of movement must be based on, and adhere to, the principles of animal movement outlined above. That is, an adequate theory must explain how movements are perceptually and intrinsically
guided. It must explain the form of the guiding perceptual information that enables prospective guidance of movement. And it must be biologically plausible. So, where do we start?

**Motion-gaps**

A key aspect of animal movement is that it is goal-directed. Therefore a basic concept is that of motion-gap. I have introduced the concept above when discussing the closure of *distance* motion-gaps, as when catching a ball. But the concept is more general than this. A motion-gap is the changing gap between the state the animal is currently in and the goal state that it wants to be in. When reaching for a fruit there is the *distance* motion-gap between the hand and the fruit. When turning gaze to look at something there is the *angular* motion-gap between the current gaze direction and the direction of the object. When thrusting off from a stair there is the *force* motion-gap between the current force and the force require for satisfactory lift-off. When singing, there is the *pitch* motion-gap between the current pitch and the next pitch – which, in turn, requires controlling other motion-gaps within the vocal system. Note that the dimension of the motion-gap is different in each of these examples, namely distance, angle, force and pitch. Thus the concept of motion-gap is not tied to a particular dimension.

All actions entail closing motion-gaps. Invariably, several motion-gaps need to be controlled at the same time. Running down stairs is an example. You need to coordinate the closure of gaze-stair and foot-stair motion-gaps if you want to get down in one piece. Controlling the closure of a motion-gap requires obtaining perceptual information about the gap and how it is closing. Motion gaps come in different dimensions (distance, angle etc). But does this mean that the perceptual information about the gaps has to come in different dimensions too? At first blush that would appear inevitable. On the other hand it would lead to a complicated system of mixed-dimensions control. Maybe evolution has found a neater solution (it usually does) and measures all motion-gaps in the same dimension. But what might that dimension be? It is unlikely to be one of the dimensions we have considered so far (distance, angle, force) because that would give one type of motion-gap (distance, say) a privileged position and so would not be a symmetrical
solution. Most likely, evolution has used the dimension that underlies the process of change of any motion-gap, namely time.

**Tau: a universal variable for controlling motion-gaps**

Can a single type of temporal variable of a changing motion-gap provide sufficient information for controlling the closure of the motion-gap? It turns out that tau can (Lee 1998). Tau of a motion-gap is the time-to-closure of the motion-gap at its current closure-rate. (To express it symbolically, suppose that at time t, the size of a motion-gap is x(t) and the rate of change of x(t) is \(\dot{x}(t)\). Then tau of the motion-gap at time t is written as \(\tau(x,t)\) and this equals \(x(t)/\dot{x}(t)\).) Note that tau is a measure on any motion-gap of any dimension (the dimension of x may be distance, angle, force etc) and the value of tau may be sensed, in principle, by any perceptual means (vision, hearing, touch, echolocation, etc). Thus, to dispel a common misconception, tau is not the inverse of the rate of dilation of an optical image, any more than gravity is the apple falling on Newton’s head. The apple falling is an example of the general principle of gravity. The image dilation is an example of the general principle of tau.

**Tau-coupling**

Let us now consider how perceiving the tau of a motion gap would benefit an animal. Here a basic concept is tau-coupling. Two taus are coupled over a period of time if they remain in constant proportion during that time. Expressed symbolically, the taus (\(\tau_k\)) of two gaps, x(t) and y(t), are tau-coupled if

\[
\tau(x,t) = K \tau(y,t)
\]

for a constant K (the coupling constant). t stands for time. The gaps themselves may be of different dimensions. As an example, consider a bat flying in to land on a perch (Fig.1a). To land properly, the bat has to control the closure of two motion-gaps simultaneously. There is the distance motion-gap, X, between the bat and the perch and the angular motion-gap, A, between the current direction of the line between the bat and the perch
Fig. 1. Tau-coupling. (a) Echolocating bat flying to a perch by keeping $\tau_A = K\tau_X$. (b) Human adult intercepting a moving target at a goal zone by keeping $\tau_{HG} = K\tau_{HT}$. 
and the direction that line needs to lie in during the final approach to the perch. The distance motion-gap, X, needs to be closed to zero in a controlled way to avoid crashing into the perch. Simultaneously, the angular motion-gap, Y, needs to be closed to zero in order to approach the landing from the right direction (remember that the bat also has to somersault just before landing so that it ends up on the perch upside-down). A film analysis of a bat performing this remarkable feat indicated that it controlled its flight by tau-coupling (Lee et al 1995). (The coupling equation was $\tau(A,t)=K\tau(X,t)$, written as $\tau(A,t)=K\tau(X,t)$ in Fig. 1a). The tau-coupling automatically ensures that gaps A and X will close simultaneously. This is because a gap reaches closure as the tau of the gap becomes zero. (As $\tau(X,t)$ becomes zero, $\tau(A,t)$ becomes zero, because $\tau(A,t)=K\tau(X,t)$, and so both A and X also become zero.) However, this is not the whole story on the bat landing. The value of the coupling constant K is also important in determining the dynamics of the movement. We shall come to this later on.

Tau-coupling also applies when intercepting something (Fig. 1b). In a recent experiment (Lee et al 2001), adults had to move a ‘hand’ cursor up a computer screen by means of a joystick so that it stopped in a goal zone just as a moving ‘target’ cursor, moving in a straight line with unpredictable speed, reached the goal zone. The relevant motion-gaps here are between hand and goal, hand and target, and target and goal. Analysis of the movement trajectories of hand and target indicated that the participants solved the task by keeping tau of the Hand-Goal gap coupled onto tau of the Hand-Target gap. (i.e., they kept $\tau(X_{HG},t)=K\tau(X_{HT},t)$, for a constant, K. The equation is written as $\tau_{HG}=K\tau_{HT}$ in Fig.1b.)

**Tau-G**

The last experiment shows how intercepting a moving object such as a ball can be achieved by coupling the tau of the motion-gap between the hand and the ball onto the tau of the motion-gap between the hand and the place of interception. Information about the ball’s motion tau-guides the hand. But what about reaching the hand out to a
stationary ball, or playing a note on a piano? In these self-guided movements, again there is the tau of the gap between the effector and its goal. However, there is apparently no other tau to couple onto to guide the movement. Or, at least, there is no other extrinsic tau to couple onto. Nonetheless, self-guided movements are well formed both spatially and temporally. Therefore, there must be some information guiding them. Might the tau of the effector/goal gap be coupled onto a (changing) intrinsic tau value generated in the nervous system - by, for instance, a patterned energy flow in the brain? If so, what form might the intrinsic tau take? We might expect that, during the course of evolution, intrinsic taus will have been assimilated by animals while moving in the environment. Since gravity has a ubiquitous influence on an animal’s movement, there are likely to be intrinsic taus that reflect the animal’s movement under gravity. One very common movement is the up-and-down motion of the body during locomotion. Therefore, there might well be an intrinsic tau that corresponds to this up-and-down motion. Such was the line of thought that led me to the tau-G hypothesis.

Tau-G is a changing tau value that, at each moment, has the same value as the tau of the vertical gap to the ground of an object, such as a ball, that is launched from the ground under gravity, reaches its zenith, and then drops down to the ground again (Fig. 2). Thus, tau-G is generated by, and could be sensed by, a running animal during each flight phase. It is, therefore, deeply rooted in the ecology of animals.

Let us now consider what would be the consequences of using tau-G for intrinsically guiding a movement. We shall, for completeness, consider the tau of a motion-gap being tau-coupled onto tau-G for the full duration of tau-G (i.e. from ‘launch’ to ‘landing’). In general, however, coupling may commence part way through tau-G’s course. As an illustration, we shall consider putting a golf ball. Let the motion-gap between the clubhead and the ball at time, t, be x(t). If tau of the motion-gap is tau-coupled onto tau-G (G₂), then \[ x(t) = K G₂(T₉, t) \], where K is a constant and T₉ is the duration of tau-G. This equation can be solved to derive the dynamic equations for the motion-gap, x(t) (Appendix 1). Fig. 3a-f shows plots of x(t) and \( \dot{x}(t) \) (the rate of change of x(t)) for three ranges of K. (The plots are useful, for example, when eye-balliong data, prior to a detailed
Fig. 2. The tau-G guide, $\Gamma(T_G,t)$. Illustrating how the equation for the guide (Appendix 1) would be generated by a ball that is launched from the ground under the gravitational acceleration of $9.81\text{ms}^{-2}$, rises to its zenith and then falls back to the ground. The ball is launched at time $t=-1\text{s}$ and lands at time $t=0\text{s}$. Thus the duration, $T_G$, of the tau-G guide is $1\text{s}$. The solid line represents the height, $y(t)$, of the ball at successive times, t. $y(t)$ increases from $0\text{m}$ at $t=-1\text{s}$ (the launch), reaches its maximum of $1.23\text{m}$ at $t=-0.5\text{s}$ (the zenith), and then decreases to $0$ at $t=0\text{s}$ (the landing). The dashed line represents the vertical component of velocity of the ball, $\dot{y}(t)$. This starts with a value of $4.9\text{ms}^{-1}$ at $t=-1\text{s}$, decreases to zero as the ball reaches its zenith at $t=-0.5\text{s}$, and then becomes negative as the ball drops, reaching a value of $-4.9\text{ms}^{-1}$ as it hits the ground at $t=0\text{s}$. The curves for $y(t)$ and $\dot{y}(t)$ are calculated from Newton’s equations, ignoring air resistance. The dotted line represents $\delta(y,t)$, the gap between the ball and the ground. $\delta(y,t)$ is calculated from $y(t)$ and $\dot{y}(t)$ using the formula $\delta(y,t)=y(t)/\dot{y}(t)$. Like time, t, $\delta(y,t)$ is measured in seconds. $\delta(y,t)$ starts at a value of zero, as the ball is launched at $t=0\text{s}$. It then increases steadily to positive infinity as the ball climbs to its zenith at $t=-0.5\text{s}$. Immediately thereafter its value switches to negative infinity, and then decreases steadily to reach zero as the ball lands at $t=0\text{s}$. The plot of $\delta(y,t)$ is the same as that of a general gravity $\Gamma$-guide, $\delta(T_G,t)$, of duration $T_G=1\text{s}$. Note that when a motion-gap, $x(t)$, is coupled onto a gravity $\Gamma$-guide through the equation $\delta(x,t)=K\delta(T_G,t)$ for different constants, K, the plots of $\delta(x,t)$ (Fig. 3g-i) have the same general shape as the plot of $\delta(y,t)$ (Fig. 2). The $\delta(x,t)$s are simply scaled versions of $\delta(y,t)$, the scaling factor being the coupling constant, K. The value of K has a more dramatic influence on the plots of $\dot{x}(t)$ (Fig. 3d-f). These vary as a function of the value of K and are quite different in shape from the plot of $\dot{y}(t)$ in Fig. 2 (except for the straight line plot in Fig. 3f which corresponds to K=1).
Fig. 3. The dynamics of motion-gaps generated by coupling onto a tau-G guide. Three ranges of value of the constant K in the coupling equation $t(x,t) = K(t(G,t))$ are illustrated. Col.1: $0 < K \leq 0.5$. Col.2: $0.5 < K < 1$. Col.3: $1 \leq K$. At time $t$, $x(t) =$ size of the motion-gap (m), $\dot{x}(t) =$ the rate of change of the size of the motion-gap (ms$^{-1}$), $t(x,t) =$ of the motion-gap (s). $|f(x,t)|$ (N) is the absolute force and $|p(x,t)|$ (W) is the absolute power required to move a mass of 1kg and generate the motion-gap. The equations generating the curves are given in Appendix 1. The bottom two plots (p,q) show how the mean absolute force (N) and power (W) required to move a unit mass and generate the motion-gap varies as a function of K. Standard deviation bars are shown.
analysis, to see whether the data might fit the tau-G hypothesis.) The three ranges of $K$ give rise to distinct types of movement. For all values $K$, the club-head starts in contact with the ball (at $x(-T_C)=0$); it then recedes from the ball until it reaches the end of the backswing (at $x(-T_C/2)=-1$); finally it moves forward and contacts the ball (at $x(0)=0$). Differences between the three ranges of $K$ occur in the vicinity of the ball (i.e., when $x(t)$ is close to zero).

When $0<K\leq0.5$ (Fig. 3a & 3d), the club-head starts at rest at the ball and moves away with increasing acceleration from a zero value; it then decelerates, reverses direction at the top of the backswing, and accelerates back toward the ball; finally, it decelerates at a decreasing rate and stops at the ball. Thus, when $0<K\leq0.5$, the movement ends with touch contact, as when reaching for something light and small. As $K$ increases from 0 to 0.5, the mean absolute force and power involved in moving the club-head increases (Fig. 3j & 3m), and the maximum velocity of the club-head decreases (Lee et al. 2003).

When $0.5<K<1$ (Fig. 3b & 3e), the club-head starts at rest at the ball and moves away with a high initial acceleration (infinite, in theory), which decreases to zero; the club-head then decelerates, reverses at the top of the backswing and accelerates towards the ball; it then decelerates at an increasing rate until it reaches its maximum deceleration, which is maintained until the ball is hit. Thus, when $0.5<K<1$, the movement ends with hard contact with the ball, with the club-head decelerating. The mean absolute force and power involved in moving the club-head are both lowered by increasing $K$ (Fig. 3k & 3n). The velocity at contact is raised by increasing $K$ (for any given maximum deceleration) or by decreasing maximum deceleration (for any given $K$); and the maximum velocity of the club head decreases as $K$ increases from 0.5 to 0.66, and then increases as $K$ increases further (Lee et al. 2003).

When $K\geq1$, the club-head starts moving away from the ball at speed, decelerates at a decreasing rate (at constant rate when $K=1$), reverses at the top of the backswing, accelerates at an increasing rate (at constant rate when $K=1$) until it reaches its maximum acceleration, and finally hits the ball at high velocity. Thus, when $K\geq1$, the movement ends with hard contact with the ball, with the club head accelerating. The mean absolute
force and power involved in moving the club head are both raised by increasing K (Fig. 3I, 3o) The velocity at contact and the maximum velocity of the club head are the same when K≥1; they are each raised by increasing K, for any given maximum acceleration, or by increasing maximum acceleration, for any given K (Lee et al. 2003).

The duration (T_G) of tau-G and the amplitude of the movement also affect the dynamics of the club head. (Duration and amplitude were assumed constant in the above.) As duration, T_G, decreases and/or movement amplitude increases, velocity at contact increases, as do absolute force and power involved in moving the club head (Lee et al. 2003).

In summary, if the tau of the motion-gap between an effector and its goal is tau-coupled onto an intrinsic tau-G, the time courses of the velocity, force and power of the movement, and the velocity of contact at the end of the movement, and are all determined by the values of three parameters: the coupling constant (K), the duration of tau-G, and the amplitude of the movement.

**Tau-g**

An earlier hypothesis about intrinsic guidance of movement from rest postulated that the tau of the motion-gap is tau-coupled onto an intrinsic tau, designated \( \tau_g \) (Lee 1998). \( \tau_g \) corresponds in value to the tau of the motion-gap to a goal of an object accelerating at a constant rate from rest to a goal. Thus a \( \tau_g \) corresponds to the second half of a tau-G. Whereas tau-G is generated by both the upward and the downward motion of a ball under gravity (Appendix 1), \( \tau_g \) is generated by just the downward motion. Thus, the \( \tau_g \) hypothesis is but a special case of the more general tau-G hypothesis. The formula for \( \tau_g \) is the same as for tau-G (Equation 1, Appendix 1) except that \( \tau_g \) only extends over the second half of a tau-G. This means that experiments supporting the \( \tau_g \) hypothesis also
support the tau-G hypothesis. There are a number of such supportive experiments, spanning a wide range of activities (Lee 1998). They indicate, for example, that newborn babies control their suction when feeding from a bottle and golfers control their swing when putting in a similar way, by using an intrinsic $\tau_g$ or tau-G (Craig & Lee 1999; Craig et al 2000).

**Tau-dot**

The earliest hypothesis about the use of tau in controlling the closure of a motion-gap was formulated in the context of braking a vehicle to stop at an obstacle (Lee 1976). The hypothesis is that, *during the deceleration phase* to an obstacle, the time derivative of the tau of the motion-gap to the obstacle (tau-dot or $\dot{\tau}$) is kept equal to a constant, $k$. If $k$ is less than or equal to 0.5, this would ensure stopping at the obstacle. Unlike the $\tau_g$ hypothesis, the tau-dot hypothesis is *not* a special case of the tau-G hypothesis. However, it turns out that the two hypotheses make numerically rather similar predictions about the final deceleration phase of a movement to the goal, which is the only phase that the tau-dot hypothesis applies to. Consequently, evidence for the tau-dot hypothesis is also evidence for the tau-G hypothesis. Such evidence spans a wide range of behaviors, including hummingbirds docking on a feeder (Lee, Reddish & Rand 1991), trampolinists somersaulting (Lee, Young & Rewt 1992) and drivers braking in a simulator (Yilmaz & Warren 1995).

**Perceiving tau**

Tau-coupling requires perceiving tau: tau is perceptible through tau-coupling. Let me elaborate on this interesting duality. It would be very useful for an evolving organism. For an animal to tau-couple the tau of a motion-gap onto the tau of another motion-gap,
or onto a tau-G, it needs to perceive the tau of the motion-gap. Fortunately for the animal, there are naturally occurring tau-couplings that make the perception of tau a relatively easy matter. This is because power laws abound in Nature and because there is a simple theorem relating power laws and tau-couplings (Lee 1992). For example, suppose we have a motion-gap, \( Z(t) \), between a person and a frontal plane containing a tree (see Fig. 4a, bottom). The sensory gap, \( r(t) \), corresponds to this physical gap, \( Z(t) \). As \( Z(t) \) shrinks, \( r(t) \) expands. In fact, simple geometry reveals that \( r(t) \) is a power function of \( Z(t) \), namely \( r(t) = Z(t)^{-1} \). The final step, involving simple calculus, shows that if \( r(t) = Z(t)^{-1} \) then \( [Z(t)] = [r(t)] \). Thus the physical tau, \( [Z(t)] \), is perceptible through the sensory tau, \( [r(t)] \).

Fig. 4b (and caption) shows how also the tau of a motion-gap in a frontal plane is perceptible through a sensory tau, by virtue of the taus being coupled. The general power-law, tau-coupling theorem is this. If a sensory gap, \( r(t) \), corresponds to a motion-gap, \( Z(t) \), and \( r(t) = CZ(t)^a \) for constants \( C \) and \( a \), then \( [Z(t)] \) and \( [r(t)] \) are tau-coupled by the equation \( [Z(t)] = a[r(t)] \). Note that the exponent, \( a \), in the power law relation, \( r = CZ^a \), becomes the coupling constant, the multiplier in the tau-coupling equation, \( [Z(t)] = a[r(t)] \).

In summary, whenever there is a sensory variable, \( r(t) \), that is a power function of a motion gap, \( Z(t) \), with exponent, \( a \), then the tau of the motion-gap, \( [Z(t)] \), is, in principle, directly perceptible from the tau of the sensory variable, \( [r(t)] \), because \( [Z(t)] = a[r(t)] \). Examples of how tau of a motion-gap could be perceived via tau-coupling in echolocation, electrolocation, and other sensory modalities, are given in Lee (1998). Examples of experiments showing that tau of a motion-gap can be directly perceived visually from the planar projection of the motion-gap - and therefore without information about the size or rate of change of size of the motion-gap - are given in Yilmaz & Warren (1995) and Lee et al (2001).
Fig. 4a. How the tau of a motion gap is specified optically through tau-coupling for forward relative linear movement of point of observation. Upper diagram (a): the optic flow field portrayed as a bundle of narrow optic cones each with its apex at the eye and its base on a surface texture element in the environment. As the eye moves forward the cones fan out. Middle diagram (b): the optic flow field portrayed as the intercept of the bundle of optic cones with a projection plane in front of the eye and perpendicular to the line of locomotion. Bottom diagram (c): A slice through the optic cone bundle containing the line of locomotion, OB, and the line, OT, to the texture element on the tree. A projection plane perpendicular to the locomotor line is added at unit distance behind O, onto which the image of T is projected, at distance r from the image of O. Tau of the motion-gap Z(t), $\tau(Z,t)$, is specified by $\tau(r,t)$ through the coupling equation $\tau(Z,t) = -\tau(r,t)$. (Proof. From similar triangles, $Z(t)/D = 1/r(t)$. Differentiating with respect to time, $t$, $(dZ(t)/dt)/D = -(dr(t)/dt)/r(t)^2$. Eliminating D from the two equations, $Z(t)/(dZ(t)/dt) = -r(t)/(dr(t)/dt)$, i.e., $\tau(Z,t) = -\tau(r,t)$.)
Fig. 4b How the tau of a motion gap is specified optically through tau-coupling for sideways relative linear movement of the point of observation. $x(t)$ is the projection of the motion-gap $X(t)$. From similar triangles, $X(t)/Z=x(t)/1$. Hence $\tau(X,t) = \tau(x,t)$. 
Distance from tau and direction

Tau is not everything by way of information needed by an animal to get around in the world. But it is almost everything. Color apart, the other information required is directional information. An animal needs to be able to perceive the direction in which something lies relative to the body, e.g., in order to be able to move toward it. Directional information is available visually by virtue of the spatial layout of the retina, acoustically through the time and intensity differences at the ears and the multiple sound reflections in the pinnae, and haptically through the articular receptors in the joints muscles and skin.

But surely, you may ask, doesn’t an animal also need distance information– about the distance of a surface, the size of an object, its speed of movement, and so on – in order to judge how hard to thrust to jump over a ditch, how wide a grasp is required to pick something up, how hard a cross-ball needs to be hit to deflect it into the net? It is true that distance information is needed, but an animal does not require a different form of sensory information for this. Information about tau and direction are sufficient (Lee 1980; von Hofsten & Lee 1994). The basic argument is this. Distance information is necessarily relative. There is no such thing as absolute distance. For distance information about the environment to be useful to an animal (whether in perceiving distance, size or speed), it has to be scaled relative to the body in some way. This means that perceiving body-scaled distance (e.g., of a surface) requires picking up information about two relative distances: the objective-distance of the surface itself and a bodily-distance, such as eye-height or stride-length. The ratio of the relative objective-distance to the relative bodily-distance gives the body-scaled distance. Information about tau and direction is sufficient for perceiving both the relative objective-distance and the relative bodily-distance. Thus body-scaled distance is derivable from information about tau and direction.

Synergetic tau-G guidance
Suppose you are guiding your gaze to an object. In general, this will involve rotating your eyes in your head and rotating your head on your shoulders, as well as twisting your
trunk and maybe shifting your feet. The question is: how do you manage to accurately guide your gaze to the target when it involves so many component movements? The same basic question as to how component movements are organized into a synergy to guide an effector to a goal applies to virtually all movements. This central question has never been satisfactorily answered, however. Here, I propose an answer, or partial answer, by applying General Theory to the problem.

According to General Theory, the focal aspect in shifting gaze, for example, is tau-G guiding the closure of the angular gap between the current direction of gaze and the direction of the object. This tau-G guidance of gaze is achieved through the combined movements of eyes, head, etc. However, each of these components also has its own agenda to follow: it has to abide by particular bodily constraints. The eyes and head, for instance, have to be turned in such a way that the muscles and joints are not strained by too abrupt accelerations or decelerations. My suggestion is that this is achieved by independently tau-G guiding the movement of each component (eye, head etc) to a goal position lying within the bodily constraints, while also tau-G guiding their combination (gaze) to its goal. I shall refer to this as the tau-G synergy hypothesis. In the tau-G synergy, the movements of the eyes-in-head, head-on-shoulders etc may or may not be tau-coupled to the gaze movement, or to each other; and the movements may or may not occur over exactly the same time period. What defines a tau-G synergy is simply that all the movements involved in the synergy are tau-G guided. Later, I shall present evidence in support of the tau-G synergy hypothesis when discussing guidance of gaze. However, before going on to the next section, I should add a caveat. I am not proposing that the component movements that move a bodily effector are invariably accurately tau-G guided, any more than the effector itself is accurately tau-G guided in every instance. Biological control is not perfect, though it can be honed with practice. Even then errors can occur, as in the famous case of the baseball pitcher who snapped his arm by hurling too hard. Indeed, it seems likely that many sports injuries are due to errors in tau-G guiding component movements within their bodily constraints.
**Stabilising with a tau-G guide**

Let us now consider the very common problem in movement control of stabilising an effector within a goal zone. By this I mean keeping the effector within the boundaries of the goal zone. For instance, the effector may be your gaze and the goal zone a moving object that you are inspecting, or the effector may be your vehicle and the goal zone the traffic lane you are driving along, or the effector may be the vertical projection of your centre of gravity and the goal zone your base of support when you are trying to stand still. Whatever the effector, stabilising it within a goal zone could be achieved by repeatedly tau-G guiding the gap, X, between the boundary of the effector and the boundary of the goal zone (such that $\int X,t=K\int (T,G,t)$, for $0<K<1$), with the tau-G guidance including some bounce-back. This would cause the effector to stop momentarily within the goal zone and then reverse direction. At which point, another tau-G would guide it to another momentary stop within the goal zone, followed by reversal. And so on. Later, I shall present evidence for this with regard to stabilising gaze on a moving object.

**Tau-G guidance in developing skill**

Skilled movement requires controlling the closure of a set of motion-gaps in a concerted manner. My hypothesis is that this is attained by coupling the tautos of the motion-gaps onto the tautos of other motion-gaps and onto tau-G guides. Thus, skilled movement comprises a balanced ensemble of tau-couplings. Controlling a tau-coupling requires constantly determining the power that needs to be pumped to the muscles to regulate the closure of the motion-gaps involved in the tau-coupling. If, for example, you are following an object with your eyes, you not only need to sense the motion-gap between your gaze and the target, you also need to know how to adjust the power to your muscles to move your gaze back onto the target when it wanders off. Thus, calibrating the process of regulating power to the muscles on the basis of prospective sensory information about the tautos of motion-gaps is an essential aspect of learning to move skilfully. Because of changing dynamical circumstances (growth, joint stiffness, injury, wearing new spectacles, etc) the calibration needs to be constantly adjusted. Recalibration experiments (e.g. Hay & Pick, 1966; Rieser et al 1995; Pick et al 1999) indicate that calibration takes place in both sensory information pick-up and in the regulation of power to the muscles.
Fig. 5. Tau information flowing around a perceptuo-motor cycle and coupling with intrinsically generated tau information.
**Tau within and without**

I shall conclude this introduction with a summary of some of the main points of General Tau Theory and then briefly discuss how tau might be embodied in the nervous system. As an illustration, consider someone playing a tune from memory on the piano. This clearly involves both intrinsic and sensory guidance (Fig. 5). According to General Tau Theory, controlling the finger movements to play expressively requires guiding the closure of the motion-gaps between the fingers and keys using information about the tau of the motion-gaps. This tau information is picked up by the perceptual systems by detecting corresponding taus in the patterns of sensory input to the eyes and/or ears and/or skin. The perceptual systems translate this tau information into neural tau information in the nervous system. Using the principle of tau-coupling, whereby one tau is kept proportional to another, this tau information, together with intrinsically generated neural tau information, in the form of tau-G guiding functions, directs the muscles to change the taus of motion-gaps in the desired way. These changes are detected by the perceptual systems and provide feedback. And so the cycle continues.

What form does tau information take in the nervous system? It must be some function of the rate of flow of electrical energy in ensembles of neurons. I shall refer to this rate of flow of energy as ‘neural power’. (In the mammalian central nervous system, neural power flows as trains of homogeneous electrical pulses (spike trains), and is often referred to as ‘neural spike-rate’. I think the term ‘neural power’ is better because it is more general and can be applied to any form of flow of information as electricity within an animal, whether continuous or discontinuous.) However, tau cannot correspond to neural power as such, because the dimensions do not match - tau is measured in time units whereas neural power is measured in power units. However, tau could be specified neurally by the *tau* of neural power (measured relative to a base level). This hypothesis was tested by analysing neural power data collected from monkey motor cortex and parietal cortex during a reaching experiment (Lee, Georgopoulos, Pepping & Lee 2003). It was found that, in each cortex, the tau of neural power relative to a base level was highly correlated with the tau of the motion-gap between the monkey’s hand and the target, and all three taus were highly correlated with a tau-G guide. Thus, the experiment
indicates that tau information is embodied in the nervous system as the tau of neural power.

**TAU IN DEVELOPMENT**

In the remainder of this chapter I shall consider how General Tau Theory might help in understanding the development of some basic skills - feeding, breathing, vocalizing, guiding the head and gaze, guiding the hand, and guiding the feet when walking and running. For the most part, each section will start with an analysis of adult skill and move on to consider how the developing child progresses toward the adult level.

**Feeding, breathing and vocalizing**

Fuelling the body is essential for movement and involves skilled movement itself. Therefore, it is not surprising that most skilled actions of young babies center around the mouth and nose. Infants start their life outside the womb with a controlled gusty outflow of air with which they announce their needs. Shortly later they are again busy with their mouths, this time regulating the inflow of milk. Crying and sucking are sophisticated skills, and babies practise them a lot. But they have pretty good command of the skills from the start - in contrast, for example, with their ability in picking up objects. This makes sense, of course, because they have to cry and suck to get food. But how do they do it?

**Tau-G guidance of nutritive sucking**

Let’s start with sucking. Sucking milk from a breast or bottle basically requires creating suction pressure in the mouth so that the milk is drawn in. This involves a number of coordinated movements in and around the mouth (Bu’Lock et al, 1990). The lips close tightly around the nipple to form an airtight seal. The jaw lowers and the tongue, which is cradling the nipple, moves forward in the mouth and hollows in the middle. The action
results in steadily increasing the size of the oral cavity and hence progressively increasing the suction. When an adequate level of suction has been reached and milk drawn into the mouth, the suction is then steadily lowered by the reverse action of raising the jaw, flattening the tongue and moving it back in the mouth. This movement also draws the tongue along the nipple and squeezes further milk from it. Not only is the whole action a wonder of coordination but newborn babies can even regulate the degree and duration of their sucks to fit with naturally occurring changes in the flow rate and fat composition of the milk (Mathew 1991, Woolridge et al 1980).

If you think this sounds as sophisticated as an adult skill such as reaching you would be right. Certainly the root elements are there. Both baby and adult are controlling motion-gaps by coordinating a number of muscles and joints. The baby is controlling a pressure motion-gap, the adult when reaching is controlling a distance motion-gap. But does it go further than that? Do babies intrinsically tau-guide their sucking movements in the same way as adults tau-guide their reaches (Lee, Craig & Grealy, 1999)? A recent study indicates that indeed they do (Craig & Lee 1999). The changing suction was recorded in the mouths of full-term newborns aged between 28 and 82 h. while they were bottle-feeding. The hypothesis, illustrated in Fig. 6a, was that the baby would continually sense, \( \tau \), the tau of the pressure-gap between the intra-oral suction and its end value; and would continually adjust its sucking pressure so as to keep \( \dot{\tau} \), coupled to an intrinsic tau-G guide (symbolized as \( \tau \) in Fig. 6), by maintaining the relation \( \dot{\tau} = k \tau \), for a constant, k. To test the hypothesis, first the pressure and its time derivative, ‘rate of closure of pressure gap’, were plotted against time. Fig. 6b, left panel, shows typical plots for a full-term infant. From these data, \( \dot{\tau} \), was calculated at each time point, for both increasing and decreasing suction, using the formula: \( \dot{\tau} = \frac{(\text{pressure minus end-pressure})}{(\text{rate of closure of pressure gap})} \). Then \( \tau \) was plotted against \( \dot{\tau} \) (Fig. 6b, right panel), using the formula for \( \dot{\tau} \); given in Appendix 1, Equation 1. Finally a linear regression analysis was run on the plot to determine the strength of the coupling between \( \tau \) and \( \dot{\tau} \) (measured by the \( r^2 \) value of the regression, which equals the proportion of variance accounted for by the linear model), and to estimate the value of the constant, k, in the coupling equation.
Fig. 6. Tau-G guidance of nutritive sucking in term and pre-term infants. (a) Diagram of the hypothesis; see text for details. (b) Full-term infant. Left: typical plots of intra-oral pressure (solid line) and its time derivative (broken line) during increasing suction (decreasing pressure) followed by decreasing suction (increasing pressure). Right: Corresponding plots, for both increasing and decreasing suction, of $\tau_p$ (tau of the gap between the intra-oral pressure and its end value) against the hypothesised intrinsic tau-G guide (symbolised as $\tau_G$ in the plot). Similar plots as in (b) but for a pre-term infant who showed very weak coupling between $\tau_p$ and tau-G. See text for more details. From Craig, Grealy & Lee (2000).
\( \text{tp}=k\text{tg} \) (measured by the slope of the regression). The mean \( r^2 \) values of 12 sucks for each of twelve full-term infants was greater than 0.95 (except for one extraneous value), indicating that the tau of suction was strongly coupled onto a tau-G guide.

Not all babies can suck so well, however. In a further study, the sucking performance of pre-term infants born at low gestation and/or birth-weight was tau-G analysed in a similar way over a four-week period (Craig, Grealy, Lee, 2000). Fig. 6c gives an example of the irregular sucking that was symptomatic of poor coupling between tau of suction (\( \text{tp} \)) and tau-G (\( \text{tg} \)). After four weeks feeding experience, only two of the six pre-term infants had reached the same strength of coupling between tau of suction and tau-G in the sucking phase as the full-term infants had within two days of birth. Furthermore, the pre-term infants who showed the most disability in sucking also fared less well in physiotherapy tests conducted (blind) around 7 months corrected age. Thus, tau-G assessment of sucking could be a useful item in the neurological examination of the pre-term infant.

**Tau-G guidance of breathing**

Babies also have to coordinate their breathing with their sucking. In another recent study (Craig, Lee, Freer, Laing 1999), it was found that term babies breathe very regularly during the pauses between sucking, indicating good coordination. However, preterm infants with bronchopulmonary dysplasia, who have difficulty in maintaining adequate levels of oxygenation of the blood during rest, had significantly less regular breathing and shorter duration breaths in the pauses during feeding. This indicates poor control of movement of the respiratory musculature, which may be an important contributing factor in bronchopulmonary dysplasia. It has not been tested whether the normal breathing of term babies is tau-G guided. It seems likely that it is, given the wide range of actions that are. However, the study needs to be done - using, e.g., an instrumented band around the chest to measure the changes in the circumference of the chest. If it does turn out that healthy infants tau-G guide their breathing, then measuring the degree of tau-G guidance in breathing could be a useful diagnostic tool that could provide useful information for devising therapies.
Fig. 7. Tau-G guidance in singing a scale legato. Top: How pitch varied with time as a female singer sang a major scale legato. Middle: A pitch-change between two notes: the top line represents pitch and the lower line represents the rate of change of pitch. The start and end points of the movement are indicated. Bottom: An example plot of tau of pitch against a tau-G guide. Data points are 1ms apart. From Fraenkel (2001).
**Tau-G guidance in vocalizing**

So far, the only study of tau-G guidance in vocalizing has been in adults singing a scale and some other musical intervals (Fraenkel, 2001). The singing was legato and so there was a smooth transition in pitch from one note to the next. Fig. 7 (top) shows pitch plotted against time for a female singer singing a major scale legato. It looks like a staircase, with the steps corresponding to the notes of the scale. Note that each main step in pitch is usually followed by some vibrato on the goal note. Fig. 7 (middle) shows an example of how pitch and its time derivative changed as the singer moved from one note to the next. The curves are similar to those for babies sucking (Fig. 6). The degree to which a movement in pitch from one note to the next was tau-G guided was determined as follows. First, the main note-to-note movement was demarcated by the times when the rate of change of pitch just exceeded zero. These times are indicated by the start and end lines in Fig. 7 (middle). Second, the tau of pitch was calculated as the tau of the gap between pitch and its value at the end of the movement. Third, tau of pitch, $\tau$, was plotted against the hypothesised tau-G guide (symbolized as $\tau$ in Fig. 7) for the movement, which was calculated using the formula given in Appendix 1, Equation 1. An example of such a plot is shown in Fig. 7 (bottom). Finally, the plot was subjected to a recursive linear regression analysis that removes the leftmost points in the plot one by one until the $r^2$ of the regression - which measures the percentage of variance accounted for by the hypothesis $\tau = K\tau$, for constant $K$ - exceeds a criterion level. The criterion was set at 0.98 in the singing study. The number of points remaining, divided by the total number in the movement, measures the % of the movement during which intrinsic tau-G guidance is in evidence. The slope of the regression at this point is taken as the measure of $K$. The mean (sd) of the measures for two singers were: $r^2 = 0.983, 0.982; % = 64, 71$; $K = 0.482, 0.495$. Thus there was strong evidence for intrinsic tau-G guidance of pitch during the last two thirds of the movement in both singers. Furthermore, their $K$ values were very similar and such that they landed gently on the next note while keeping mean force and power low.

Do infants tau-G guide the pitch, timbre or loudness of their voice when crying, cooing, babbling etc? Do adults do likewise in speaking? We do not know, because the
experiments have not yet been run. This whole area needs to be explored. It could help us understand better how vocalizations are produced, and this could be of value in diagnosis and treatment.

Guiding head, eye, and gaze

This is of vital importance for an animal in order to pick up accurate information through vision, hearing and smell about objects and events and about the movement of the animal relative to the environment. Without this ability, perceptual guidance of bodily movement would be severely compromised. In a series of elegant experiments reported in this volume, Claes von Hofsten and Kerstin Rosander show how an infant develops the ability to coordinate the movements of eyes-in-head and head-on-body to keep gaze on a moving target. Here, I would like to develop this theme by taking a general look at head and gaze control.

Let’s start with a rural scene. One day, as you are walking through the woods with your dog, you hear a twig snap behind you in the undergrowth. You both turn around to the source of the sound to see what it is. Out of the corner of your eye you catch sight of something moving. You swing your gaze after it and see a squirrel bounding away. Your dog, of course, is off like a shot after the squirrel, leaping over branches in hot pursuit. But the squirrel easily avoids capture by running up a tree trunk.

This commonplace scenario illustrates how the head is guided for the purpose of perception. When you hear the snapping twig, you and your dog both turn around to point your heads at the sound source to pick up more information about the event through vision, hearing and smell (particularly for your dog). You perceive the direction of the sound relative to the head, then use this information, together with on-line perceptual information about the head when it is rotating, to guide it to point it at the goal. The on-line information about the rotation of the head relative to the environment is available through vision (by seeing how the head is rotating relative to the environment), through articular proprioception (by feeling how the head is turning relative to the ground) and through the vestibular system (by sensing the acceleration of the head relative to the
inertial environment). If, however, firm contact with the ground is not maintained while
turning (as would apply, e.g., to fish or birds in currents), then vision and the vestibular
system would be the only sources of on-line information for controlling the movement of
the head.

While your head and body are still turning, you catch sight of the squirrel. You
therefore swivel your eyes in your head to latch your gaze onto it, using visual
information. You now need to keep you gaze on the squirrel, and since it and your head
are moving, you have to keep your eyes moving in a controlled way in your head, again
using visual information. You may also have to modify your initial planned movement of
your head to cope with the emerging situation. Meanwhile, your dog has got off to a
flying start and you turn your gaze to it. You notice how steadily it moves its head over
the ground, keeping it coupled onto the locomotor optic flow field (Fig.4). This is in
contrast to the rest of its body, which is engaged in varied vigorous movement to propel it
over the rough ground. Also, its head is now about to be used as an instrument of action.
But before it can close its jaws the squirrel escapes. (Moving the head with precision as
an effector is not, of course, the prerogative of aggressive acts. Birds do it when they are
feeding their young, and we do it when eating and kissing.)

**Tau-G guidance of head, eye and gaze**

If we want to explain how the gaze (or head) is guided to a target and kept on the target,
then we need an answer that is applicable to the general kind of situation just described.
This is not only because that is of interest for its own sake but also because in more
circumscribed conditions the same guidance procedures are likely to be used. Otherwise,
as conditions changed, the procedure would constantly have to change. To reiterate, *the
general situation in shifting gaze is when the target object is moving in the world, the
eyes are turning in the head, the head is turning on the body and the body is turning and
translating relative to the world.*

The presiding motion in this plethora of movement is the closing of the gap between
gaze line and the target. The synergic motions of eyes-in-head, head-on-body etc are
largely incidental, in the sense that they can be executed in a variety of combinations to
achieve the same end of guiding gaze to the target and holding it there. However, this
Fig. 8. Tau-G guidance of gaze, eye and head. (a) Experimental setup. (b) Plots, from a typical trial, of gaze direction relative to target direction (solid line), eye direction relative to head direction (dotted line), and head direction relative to world (broken line). (c) The time derivatives of the plots in (b). $G_0$, $E_0$, $H_0$ mark the starts, and $G_1$, $E_1$, $H_1$, mark the ends of the principal movements of gaze, eye and head.
does not mean that the synergic motions of eyes-in-head, head-on-body etc are not themselves guided. Each has to be guided to stay within body constraints. Earlier in this chapter, I suggested an hypothesis on synergic tau-G guidance of gaze. According to the hypothesis, gaze is tau-G guided to the target, while the synergists are each independently tau-G guided to goal positions within bodily constraints, in general over different time intervals. If the hypothesis is correct, the following tau-couplings are predicted:

\[ X_{G/T},t=K_{G/T} \hat{p}(T_{G/T},t) \] for Gaze/Target, \[ X_{E/H},t=K_{E/H} \hat{p}(T_{E/H},t) \] for Eye/Head, and \[ X_{H/B},t=K_{H/B} \hat{p}(T_{H/B},t) \] for Head/Body. The durations, \( T_{G/T} \), \( T_{E/H} \), and \( T_{H/B} \) may be different.

The hypothesis was tested on adults (Lee, Grealy & Schogler, in preparation). The participant started a trial by turning their head and eyes to look at a fixation point over their left or right shoulder. Then, at a signal, they swung their gaze forward to look at an object that was moving on a horizontal track at head height in their frontoparallel plane (Fig. 8, top). The bottom two panels in Fig.8 show typical plots for one participant of the Gaze/Target, Eye/Head and Head/World angles and angular velocities. The initial (principal) movements of gaze, eye, and head were taken to start when the velocity just exceeded 5\% of its peak value on the particular movement, and to end just before the velocity dropped to 5\% of its peak velocity. The start and end points are marked \( G_0, E_0, H_0 \) and \( G_1, E_1, H_1 \) respectively in the bottom panel of Fig. 8. The three movements were tau-G analysed for ten trials in the way described above for singing. This gave three measures for each movement: (1) the \% of the movement that was tau-G guided to the criterion of \( r^2 \geq 0.95 \), (2) the value of \( r^2 \), and (3) the regression slope, \( \hat{K} \), which estimates the \( K \) value in the tau-coupling equation. The initial (principal) movements of Gaze/Target, Eye/Head, Head/World were all very strongly tau-G guided (mean \% = 99\%, mean \( r^2 = 0.976 \)). The \( \hat{K} \) values were quite similar: 0.53±0.08, 0.58±0.05 and 0.49±0.11, for Gaze/Target, Eye/Head, Head/World, respectively. The values indicate low force and power in the movements. The tau-G guides were not the same, however. They started at slightly different times (the Eye/Head guide started 7±5ms before the Gaze/Target guide and 43±21ms before the Head/World guide), and their durations were
largely different (331±26ms for the Gaze/Target tau-G guide, 274±22ms for the Eye/Head guide, and 535±59ms for the Head/World guide).

Each initial (principal) movement of Gaze/Target, Eye/Head, Head/World was followed by a series of stabilizing movements (Fig. 8). The first stabilizing movements were tau-G analysed in the same way as the principal movements that preceded them. Each stabilizing movement was very strongly tau-G guided (mean % = 96%, mean $r^2 = 0.974$). The range of K values was slightly broader than for the principal movement - 0.602±0.258, 0.400±0.162 and 0.581±0.209, for Gaze/Target, Eye/Head, Head/World respectively – but, again, all indicate low force and power in the movement. As with the principal movements, the tau-G guides for the stabilizing movements started at different times and were of different durations (89±26ms for the Gaze/Target, 214±51ms for the Eye/Head, and 308±150ms for the Head/World). These results thus support the hypothesis, described above, that tau-G guidance is used in stabilizing on a target.

**Development of tau-G guidance of head, eye and gaze**

This has not yet been studied. It needs to be. For example, it would be valuable to analyse, in terms of the tau-G guide hypothesis, the movements of gaze, eye and head when shifting gaze to a moving target, as in the experiment above, and when tracking (stabilizing on) a moving object (von Hofsten, this volume). This could provide important information about the development of a vital skill. The information gained could be used diagnostically to pinpoint problem areas and suggest possible ways of aiding development.

One important aspect of the development of tau-G guidance of gaze is mastering the gaze synergy. Guiding gaze to a target while independently guiding the eye in the head and the head on the body, each with respect to its own movement boundaries, must pose quite a problem for a young baby, particularly because the head is relatively so massive. A strategy that babies appear to adopt is stabilizing gaze mainly by moving the head. This would mean that the eyes moved less in the head and so the problem of guiding them with respect to bodily constraints would be reduced. Therefore the gaze synergy would be simplified. Evidence for the increasing use of head movement comes from a longitudinal study of 11 to 29 week old infants who (a) visually tracked a toy moving in an arc in
front of them and (b) stabilized gaze on the toy when it was stationary and they were rotated to and fro (Daniel & Lee, 1990). It was found that the proportion of the gaze rotation relative to the body that was due to head movement increased from around 60% to around 90% from 11 to 17 weeks of age, and then stayed about constant until 29 weeks of age. By contrast, adults in the same experiment had a head/gaze ratio of about 60%. Von Hofsten and Rosander (1997) found a similar developmental trend in increased head movement between 3 and 4 months of age.

Guiding the hand

The hand is a versatile organ and can be guided in numerous ways, some of which are basic and develop naturally, such as reaching and grasping, while others are learned, such as writing and playing a musical instrument. Here I shall concentrate on some basic skills. They probably contribute to the foundation on which learned movements are built.

Tau-G guidance of the hand to an external goal

I shall start with an experiment that was designed to test the tau-G guide hypothesis. Adults were asked to perform three reaching tasks that involved moving the tip of a pointer horizontally within a 9cm zone to a target point on a table at the edge of the zone (Lee, Grealy, Pepping, Schogler, Craig & Georgopoulos submitted). The tasks were designed to test three aspects of the hypothesis. They were: (1) ‘forward and stop’ – start within the zone, move the tip of the pointer to the target and stop at it; (2) ‘retreat, forward and stop’ – start at the target, move away from it, stop within the zone, immediately reverse direction back to the target and stop at it; (3) ‘retreat, forward and bounce back’ – as task (2) but bounce back from the target before stopping within the zone. The participants performed each task repetitively for 30s. The motion of the tip of the pointer was recorded at 300Hz by a Selspot motion-capture system. The participant
Fig. 9. Test of the tau-G guide hypothesis on three reaching tasks with a pointer. The tasks are labelled at the tops of the columns. Results are for one participant over twelve trials. (a-c) Plots of gap between pointer tip and goal position. (d-f) Time derivatives (velocities) of the gaps. (g-i) Taus of the gaps between the pointer tip and its end position. (j-l) Plots of the taus of the gaps against tau-G guides.
stood at the side of the table while performing the tasks. Thus the reaching movements involved movements of joints and muscles throughout the body, as do most everyday reaching movements.

Fig. 9 rows 1-3 (panels a-i) shows how the gap between the pointer tip and its stopping point, \( r(t) \), the time derivative of the gap, \( \dot{r}(t) \), and the tau of the gap, \( \tau(r,t) = \frac{r(t)}{\dot{r}(t)} \), changed over time during the three types of movement. The data are for one of the six participants. The other participants’ results were similar. It will be seen that the graphs in Fig. 9 are similar to the latter sections of the theoretical graphs of \( x(t) \), \( \dot{x}(t) \) and \( \tau(x,t) \) in Fig. 3a, d, g. Thus coupling of \( \tau(r,t) \) onto a tau-G guide is indicated. To test the hypothesis we plotted each empirical \( \tau(r,t) \) against its tau-G guide (computed from Equation 1, Appendix 1), from the start to the end of the movement. For the movements involving initially retreating from the goal, the start was taken to be when the velocity of retreat reached its peak value. For the bounce-back movement the end was taken to be when the bounce-back velocity reached its peak. Sample plots are given in Fig. 9j, k, l. From the plots we calculated the linear regression coefficients, \( r^2 \) and \( \hat{K} \). \( r^2 \) is the proportion of variance accounted for by the tau-coupling model, \( \tau(r,t) = \hat{K}\tau_g(T_G,t) \), and measures the strength of the tau-coupling. We found the \( r^2 \) were high (overall mean 0.97) for all three reaching movements – ‘forward and stop’, ‘retreat, forward and stop’ and ‘retreat, forward and bounce back’. This supports the hypothesis that the movements were guided by coupling the tau of the motion-gap to the stopping point at the target onto a tau-G guide, through the equation \( \tau(r,t) = \hat{K}\tau_g(T_G,t) \). \( \hat{K} \), the regression slope, provides an estimate of the value of the coupling constant, \( K \). \( \hat{K} \) was close to 0.5. This meant there was touch contact with the goal, and the peak force and power involved in the movement was low (see Fig. 3 and discussion of Fig. 3 in the text).
Fig. 10. Tau-G guidance of the hand to the mouth. (a) Showing the position of the Selspot leds and the taus used in the analysis. (b) Typical paths of the hand to the mouth. From Lee, Craig & Grealy (1999).
**Tau-G guidance of the hand to a bodily goal**

In the preceding experiment, the pointer was tau-G guided to an *external* goal. This involved using visual information about the tau of the gap between the pointer tip and the external goal. Are movements that use only articular information (derived from sensors in the joints, muscles and skin) also tau-G guided? The following experiment tested this by requiring adults to move their fingers to a bodily goal with eyes shut. The task was to raise pieces of food from their lap to their mouth (Lee, Craig & Grealy, 1999). They positioned their head differently on each trial to avoid the possibility of making stereotyped movements and to make sure that they had to perceptually guide their hand through articular information. The movement of the fingers to the mouth was recorded at 312Hz on a Selspot motion-capture system. Fig. 10 illustrates the set-up and the measures computed from the Selspot records. It was found that the gap, \( r(t) \), between the hand and the mouth was closed by keeping \( \tau(r,t) = K_{\tau}(T_G,t) \) (in Fig. 10, \( \tau(r,t) \) is written as \( \tau \)). The coupling was tight (mean \( r^2 > 0.985 \) for on average 87% of the movement). Also, the hand had to arrive at the mouth from the right direction to avoid, e.g., colliding with the nose. This involved controlling the angle \( \alpha \) (Fig. 10), the steering component of the movement, and was achieved by tau-coupling \( \alpha(t) \) onto \( \tau(r,t) \) by keeping \( \alpha(t) = K_{\alpha} \tau(r,t) \), for a constant, \( K_{\alpha} \). Again the coupling was tight (mean \( r^2 > 0.980 \) for on average 82% of the movement). Virtually identical results were obtained when the task was performed with eyes open. Thus the articular information (from sensors in the joints, muscles and skin) about the movement of the hand to the mouth that was used in guiding it was as precise as when visual information was added.

**Development of tau-G guidance of the hand**

How does the ability to tau-G guide the hand develop? I shall consider reaching to external and bodily goals separately because they appear to develop differently. Whereas adults can reach to either with about equal facility, young infants appear to find bodily
Fig. 11. (a) 10-week-old baby girl being settled to hear some music. The Selspot and video cameras in the foreground recorded her limb movements. (b) Typical movement profiles of the 10-week-old infant’s right foot. The vertical lines indicate the start and end points of the tau-G guidance analysis. From Hooker (2003) and Perkins (2003).
goals easier. It is a common observation that young infants manage to get their hand to their mouth, but they make a poorer showing of reaching out for toys. In fact, infants do not start reaching with any skill for external objects until they are four or five months old (von Hofsten, 1983). Thus infants under this age would appear to be not as capable as adults at tau-G guiding their hands to external objects.

Are infants capable of tau-G guiding their hands to bodily goals? Evidence that they are comes from a recent experiment carried out by Hooker (2003) and Perkins (2003). The original purpose of the experiment was to study how young infants move to music. A 10 week old little girl was supported on a special rest (designed by Colwyn Trevarthen) that simulates being held against the shoulder, while allowing free movement of the arms and legs (Fig. 11). Selspot leds were attached to her hands and feet and her movements were recorded at 125Hz as different types of music (jazz, funk, and Indian tabla) were played to her through a loudspeaker. During the concert, which lasted about half an hour and included some quiet periods, she moved her hands up and down and swung her feet sideways. The vertical movements of her right hand and the rightwards movements of her right foot (Fig. 11) were analyzed in a similar way to that described above under ‘tau-G guidance of vocalizing’, but using an \( r^2 > 0.95 \) criterion. The movements were generally found to be strongly tau-G guided. For the rightward movement of her right foot, the mean \( r^2 \) across the four conditions was 0.98 for on average 97% of the movement. For the vertical movements of her hand, across the four conditions the mean \( r^2 \) was 0.97 for on average 97% of the movement. There was no significant difference in the degree of tau-G guidance across the four conditions, as indexed by the \( r^2 \) and the %. The mean \( \hat{K} \) values for both her foot and hand movements were mainly around 0.6 (range 0.57 to 0.66), which means that she kept the peak force and power in the movement relatively low and brought the movements to a stop quite gently. An exception to this was that the mean values of \( \hat{K} \) for her hand with the funk and tabla music were significantly higher at 0.98 and 0.96. Here she was applying more force and power and bringing her movements to a stop more abruptly. Thus not only did the experiment provide strong evidence of tau-G guidance of the hand and foot to bodily goals at 10 weeks of age, it also provided
Fig. 12. Visual prospective guidance of catching. Upper: An attractive toy moves on a straight track from the infant’s left or right, enticing the infant to reach to catch the toy through the central window, which has occluders either side of it. Lower left: typical plots for a 20 week old infant, with the toy moving at 8 cm s\(^{-1}\), of the y-coordinate of the hand (thin line) and the x-coordinate of the toy (thick line) as recorded by an overhead camera with its optical axis vertical and the x-axis of the image plane parallel to the toy’s motion path. Lower right: typical plots for the same infant at 40 weeks of age, with the toy moving at 6.5 cm s\(^{-1}\). The open arrows show when the hand begins to move forward and the closed arrows show when the toy is caught. The first interruptions in the record of the motion of the toy are due to the toy being behind the occluder. By 40 weeks of age the hand showed prospective control: it started to move before the toy went out of sight behind the occluder and at a time that was more related to the tau of the gap between the toy and the catching place than to the size of the gap. From van der Meer, van der Weel & Lee (1994).
Fig. 13. Tau-coupling in hitting. Upper: The ball is rolling down the track toward the nursery school boy. He is going to try to hit a bat hanging to his right of the track just as the ball reaches the bat. If he succeeds, the ball will knock over the cans to his left, causing great glee. Lower: Plots showing the coupling between tau of the gap between the hand and the bat ($\tau$-Hand) and tau of the gap between the ball and the bat ($\tau$-Ball). From van der Weel, van der Meer & Lee (1996) and Lee, von Hofsten & Cotton (1997).
evidence suggesting that the baby was modulating the expressive form of her movement by regulating the value of the coupling constant, $K$, in the tau-G guidance.

If a young baby of ten weeks of age can tau-G guide her hand and foot to goals within her body-space, with or without music, why can she not tau-G guide her limbs to external goals with equal accuracy? She will start reaching for objects only around 16 to 20 weeks of age. Even then her movements will be nowhere near as smooth and well-formed as her movements to bodily goals at 10 weeks. Her reaching movements will initially consist of a number of alternations of acceleration and deceleration, like a driver switching back and forth between the accelerator and brake. As she gets more proficient, the number of switches between acceleration and deceleration will decrease until eventually, by about 31 weeks of age, she is reaching with a single acceleration phase followed by a single deceleration phase - with possibly one or two small acceleration/decelerations added at the end if she needs to zero-in precisely on the object (von Hofsten, 1991). During this period she will also get increasingly more proficient at catching moving objects (von Hofsten 1983) and by about 40 weeks she will start using tau in prospectively initiating her catches (Fig. 12). Later she will use tau-coupling in hitting a moving ball (Fig. 13).

So, why can a young baby tau-G guide her hand to a bodily goal so much better than she can to an external goal? When the baby is tau-G guiding her hand to a bodily goal, she is continually sensing the tau of the gap between her hand and the goal through articular sensors in her muscles, joints and skin (possibly supplemented by vision). She is regulating the force exerted by her muscles so that the tau she is sensing stays in constant ratio with the tau-G she is generating in her nervous system. In the womb, she will have been able to practice obtaining information through her articular sensors to guide her hands to bodily goals such as her mouth. Therefore, by the time she is born, she could be quite adept at guiding her hand to bodily goals. On the other hand, when she is guiding her hand to an external goal, she has to use visual information about the tau of the gap between her hand and the goal. Since she cannot use her eyes in the womb, when she is born she has quite a bit of catching up to do with regard to visual guidance. She needs to learn how to pick up tau information visually and put it into action.
Fig. 14. Upper: A newborn baby moving its hand in a darkened room into a light beam (presumably to see the hand better). Lower: typical position (thin line) and velocity (thick line) record of the hand measured parallel to the body axis. After time 15.6s the hand was in the light beam. Note how the hand moves smoothly into the light beam and then stays there. From van der Meer (1997).
A good starting point would be for her to watch her hands moving, so that she can both see and feel the same movement and start to get the two senses in registry. Newborns will work to do this, resisting pulls on their hands, even when they can only see their hand in a TV monitor (van der Meer, van der Weel & Lee 1996). Audrey van der Meer (1997) investigated this further. The newborn baby lay on its back facing sideways (Fig. 14, upper). The room was dark except for a narrow beam of light that shone across the baby’s chest in the direction she was facing. She could see her hand clearly when she raised it into the beam, but only dimly otherwise. A Selspot led was attached to the baby’s hand to record its movement. Fig. 14 (lower) shows a typical record of hand movement. The hand moves smoothly into the light beam and stabilizes there. Her movements were not tau-G analyzed and so we do not know to what degree the baby was tau-G guiding them. However, the position and velocity curves in Fig. 14 (lower) are similar to the tau-G theoretic gap and gap velocity curves in Fig. 3 b & e (apart from the curves being inverted). Therefore, it is likely that a tau-G analysis of the data would reveal that newborns tau-G guide their hands into the light beam. This would imply that young infants are capable of visually tau-G guiding the hand - even though they appear not to be when they are presented with an object to reach out for. This apparent disparity could be explained by the different optic flow fields in the two situations. When moving the hand into the beam, the hand moves approximately in a plane perpendicular to the direction in which the infant is facing. Thus the basic optic geometry is as shown in Fig. 4b. On the other hand, when reaching forward for an object, the hand moves approximately in the direction in which the infant is facing. In this case, the optic geometry is similar to that for forward locomotion (Fig. 4a).

Guiding the feet

As anyone who has traversed uneven ground knows, walking and running are not regular oscillatory processes like those driving a clockwork soldier, despite what some texts assume. Indeed, flesh and blood soldiers have to train hard to emulate the regularity of
Fig. 15. Tau-G guidance of the foot from one point of contact to the next when sprinting. The motion-gap is between the foot and the upcoming point of contact.
their clockwork counterparts. No, walking and running are far more akin to reaching with the hand than to oscillating. The feet have to be guided to suitable points of contact with the ground from which the body can thrust off to the next point of contact. The optic flow field provides information for this (Fig.4a). Watch a young child at the beach climbing barefoot on rocks and you will see what I mean by reaching with the feet. You will also see toes in a new light, as little fingers gripping the rock.

**Tau-G guiding the feet when sprinting**

Viewing walking and running in this way naturally leads to the question: Are the feet tau-G guided like the hands are? To answer this question, we recorded (at 250Hz, using Selspot) the limb movements of sprinters running on an indoor track (Lee, Grealy, Pepping & Rewt, in preparation). The first motion-gap that we examined was between the toe and the up-coming point of contact with the ground (Fig. 15). This motion-gap is clearly important for general running, to secure adequate footing. To be sure, running on a track is not the same as running over rough ground. However, it seems highly unlikely that runners will adopt different modes of control depending on the nature of the ground. They are far more likely to have a single flexible mode of control that they fit to the terrain.

One important thing a runner has to strive for is attaining proper *dynamical* contact with the ground. The foot not only has to arrive at an adequate place, it has to be moving in an efficient way as it arrives. Suppose, for example, the ground is horizontal, like a running track. First, if the foot arrives at the ground while it is still moving forward relative to the ground, this will produce a retarding force on the body. This is, in fact, what runners do if they want to stop, but it is not what a sprinter wants to do when in full flight. Second, if the foot arrives at the ground while it is still moving downward then this will produce a jarring vertical impact with the ground. The vertical impact appears to be minimised in good sprinters. Their footsteps are quiet as they glide over the ground.
Fig. 16. Tau-G guidance of the foot horizontally *relative to the hip* when sprinting. (a) When the foot is moving backwards, the relevant motion-gap is the horizontal distance-gap between the current position of the toe and the rearmost position it will reach. (b) When the foot is moving forwards, the relevant motion-gap is the horizontal distance-gap between the current position of the toe and the foremost position it will reach. In both (a) and (b), the horizontal distance between the toe and hip are shown as horizontal lines. The length of this line in the rightmost drawings is the ‘goal distance-gap’, when the corresponding motion-gap is closed.
A second important thing a runner needs to strive for is moving the limbs efficiently. Taking this point into account, together with the preceding points, we predicted that the toe would be tau-G guided to the ground with a coupling constant, K, which ensured low impact velocity with the ground while also keeping power consumption low. Now, mean power consumption is lowest when 0.5<K<1, and it slowly decreases as K increases from 0.5 to 1.0 (Fig.3p,q). On the other hand, for 0.5<K<1, impact velocity increases quite steeply as K increases from 0.5 to 1.0 (Fig.3e). Therefore, keeping power and impact velocity both low should entail a K value at the lower end of the range from 0.5 to 1.0. This is, in fact, what we found. For four sprinters, running at speed, the motion-gap between the toe and the ground was tau-G guided, with an $r^2>0.95$, during 99% of the movement on average, with mean K values ranging from 0.51 to 0.61.

It would be little use getting your foot accurately placed on the ground if, at contact, it were not also accurately placed horizontally just in front of the hips, so that the leg can drive the body forwards and upwards. Thus it is also important to control the horizontal movement of the toe relative to the hip. We found this, too, was achieved using tau-G guidance by the sprinters we analysed. During the backward movement of the toe, the tau of the motion-gap between the toe and its rearmost position (rightmost drawing in Fig. 16 a) was coupled onto a tau-G, with an $r^2>0.95$, during 99% of the movement on average. The mean K values ranged from 0.47 to 0.52. During the forward movement of the toe, the tau of the motion-gap between the toe and its forward-most position (rightmost drawing in Fig. 16 b) was coupled onto a tau-G, with an $r^2>0.95$, during 73% of the movement on average. The mean K values ranged from 0.58 to 0.77. Thus, as with the tau-G guidance of the toe relative to the ground, both power and final velocity were kept low when moving the foot relative to the hip.

**Tau-G guiding the feet down a slope**

Running and walking up and down slopes emphasizes the importance of properly guiding the feet relative to the ground and the hips. The steeper the slope, the more precise is the control required. One reason for this is that the horizontal extent of the area of contact with the ground - the base of support - is smaller when walking down a slope compared
with horizontal ground. Thus balance is more difficult (Adolph 2002, and this volume). In a series of elegant naturalistic experiments Karen Adolph (1997) has studied how infants develop the ability to perceive which slopes can be safely traversed and which are too steep. In making the perceptual judgment, the infants have to assess the slope in terms of the degree of control they could muster to negotiate it. They have to perceive the affordance of the slope relative to their current action capabilities. The degree of control required to guide the feet to walk safely on a slope could be measured in terms of tau-G guidance of the feet in the following way.

Imagine you are running down a steep slope. You have to keep pushing back on each footfall to stop yourself tippling forwards. To prepare yourself for each push, your foot should be moving forward relative to the ground at contact - but not by too much, else the force on your leg could strain your joints and muscles. The situation is, therefore, different from when sprinting at full speed, when your foot should be stationary relative to the ground at contact. In terms of tau-G guidance of the foot relative to the ground, this means that when you are going down a steep slope your foot needs to be tau-G guided with a K value that is greater than 0.5, so that there is some forward velocity of the foot relative to the ground at contact. The same applies when you are walking down a steep slope. The only difference is that for part of the time both feet are on the ground. Therefore the trailing foot can also exert some backward force on the body, to resist the accelerative force down the slope due to gravity.

With regard to the movement of the foot relative to the hip when going down the slope, this probably has to be such that the foot strikes the ground a little further in advance of the hip (measured horizontally) than it does when sprinting on a horizontal track, in order to be able to develop adequate force backward on the body to balance the gravitational pull.

In sum, running or walking down a slope probably involves modulating the K value for the tau-G guidance of the foot relative to the ground, in particular shifting K away from 0.5 and towards 1.0, in order to increase the velocity at contact. At the same time the movement of the foot relative to the hip probably needs to be adjusted in order to achieve an appropriate position of the foot relative to the hip as it touches down. Both adjustments need to be tuned to the angle of the slope. I say probably because the
experimental measurements still need to be made, with adults and young children. It would be a most valuable project to undertake, one that should increase our understanding of what is involved in negotiating uneven ground and how the ability develops.

HELPING DEVELOPMENT

Children need help in developing movement skills. The need is particularly acute in those who start life in weak physical or neural health. While there have been great advances in the ability of medical science to save the lives of babies born very prematurely, unfortunately a number of such infants grow up to be disabled. Improving the help given to infants and children in learning to guide their movements is likely to lessen the disabilities. This optimism is justified by a number of cases where infants severely neurologically damaged have nonetheless succeeded in attaining near normal function. The nervous system is apparently able to adjust to damage, providing the child has adequate environmental and human support in developing movement skills.

What is needed to help infants, children and adults develop movement skills is a set of sensitive measures of movement control that tap into the basic workings of movement control. This is necessary both for accurately diagnosing problems and for monitoring progress during therapy or training. Another need, of course, is for therapeutic or training exercises that are specifically tailored to address the problems that the tests have revealed. General Tau Theory has something to offer here, I believe. First, it provides a precise way of describing a basic component of movement control - the tau-G guidance of closure of motion-gaps. Second, it offers a way of analyzing how the closures of different motion-gaps are coordinated together into a single act, as for example in sprinting. Third it provides a precise way of measuring how well the different components are executed and how well the coordination is executed. Fourth, it provides a method for analyzing movement skills to reveal unsuspected degrees of coordination, as, for example, was found in infants sucking milk from a bottle. Fifth, it suggests
procedures that could be developed to help infants, children and adults overcome difficulties in executing different components of tasks.

In this chapter I have described a number of studies that indicate the direction in which future research directed towards helping master movement skills might proceed. Further experimental studies need to be carried out to increase our understanding of skill development. In parallel, diagnostic tests need to be developed and tested. Likewise, therapeutic and training procedures need to be devised and evaluated. This will be a large undertaking, too large for any single laboratory to carry out, but the outcome could bear much fruit. Readers who are interested in taking up the challenge and would like more information about applying General Tau Theory should visit http://www.perception-in-action.ed.ac.uk/.

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References


Appendix 1

Derivation of the equation for the tau-G guide, $\tau_G(T_G,t)$.

Imagine a ball launched vertically upwards from the ground, reaching its zenith and dropping back to the ground. Suppose $u$ is the velocity at take-off, $g$ is the gravitational acceleration and $T_G$ is the flight duration. Ignoring air resistance, $u=gT_G/2$, and the height of the centre of gravity of the ball above the ground at time $t (<0)$ before landing at time $t=0$ is $y(t)=-ut-gt^2/2$. Eliminating $u$ between these equations we obtain $y(t)=-(T_G+t)gt/2$.

Differentiating this equation with respect to time gives $\dot{y}(t)=-(T_G+2t)g/2$. Hence

$\tau(t)=y(t)/\dot{y}(t)=y(t)/(T_G+t)/(T_G+2t)$. By definition, $\tau(x,t)$ is equal to the tau-G guide, $\tau_G(T_G,t)$. Thus, writing $t_n=t/T_G$,

$$\tau_G(T_G,t)=T_G(1+t_n)tn/(1+2tn) \quad (1)$$

for $-T_G\leq t\leq 0$, or $-1\leq t_n\leq 0$. (The equation for the special tau-G guide, $\tau_s$, is the same as equation (1), the only difference being that $\tau_s$ runs over the time interval $-T_G/2\leq t\leq 0$ rather than $-T_G\leq t\leq 0$.)

Suppose that the tau function of a variable $x$, $\tau(x,t)$, is coupled onto a tau-G guide, $\tau_G(T_G,t)$, so that $\tau(x,t)=K\tau_G(T_G,t)$ for a constant $K$. From equation (1)

$$\tau(x,t) =x(t)/\dot{x}(t)=KT_G(1+t_n)tn/(1+2tn) \quad (2)$$

We can now derive the motion equations for $x$. Integrating equation (2) with respect to time, $t$, gives

$$x(t)=x_m2^{2/K}(t_n-t_n^2)^{1/K} \quad (3)$$
where \( x_m (\leq 0) \) is the minimum value attained by \( x \) (when \( t_n = -0.5 \)). Differentiating equation (3) successively with respect to time gives

\[
\dot{x}(t) = \left(\frac{x_m}{T_G} \right) (2^{2/K}/K) (-1 - 2t_n) (-t_n - t_n^2)^{(1/K-1)}
\]  
(4)

\[
\ddot{x}(t) = \left(\frac{x_m}{T_G^2} \right) (2^{2/K}/K) [(4 - 2K)/K] [t_n^2 + t_n + (1 - K)/(4 - 2K)] (-t_n - t_n^2)^{(1/K-2)}
\]  
(5)

Next we derive the force, \( f(x,t) \) and power, \( p(x,t) \), required to move a unit mass in accordance with equations (3), (4) and (5). \( f(t) = \ddot{x}(t) \) and so, from equation (5),

\[
f(x,t) = \left(\frac{x_m}{T_G^2} \right) (2^{2/K}/K) [(4 - 2K)/K] [t_n^2 + t_n + (1 - K)/(4 - 2K)] (-t_n - t_n^2)^{(1/K-2)}
\]  
(6)

\[
p(x,t) = \text{rate of change of kinetic energy} = \frac{d}{dt}(\dot{x}(t)^2/2) = \dot{x}(t) \ddot{x}(t)
\]  
(7)

where \( \dot{x}(t) \) and \( \ddot{x}(t) \) are given by equations (4) and (5).

Equations (2), (3), (4), (6) and (7) are plotted in Fig. 3 a-o for a complete cycle of \( f(T_G,t) \), i.e., for \( -T_G \leq t \leq 0 \). Values of \( T_G = 1 \)s, \( x_m = -1 \)m and mass = 1kg are used in the graphs.